## Moral-hazard premium

Takashi Misumi Hitotsubashi University Hisashi Nakamura\* Hitotsubashi University Koichiro Takaoka Hitotsubashi University

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## Abstract

We provide an equilibrium asset-pricing formula under moral hazard, which is defined as a firm's strategic change of measure that is incontractible, on the assumption of power utility and an endogenous riskless rate. Moral hazard distorts equilibrium asset prices: the market price of diffusive risk is twisted in the opposite direction of an investor's marginal utility; the market price of jump risk is amplified; a positive premium, called a moral-hazard premium here, is stipulated on a riskless rate. Thus the risk-free rate puzzle is more serious under moral hazard. Also, financial markets alleviate the allocation conflict caused by moral hazard.

Keywords: moral hazard, asset prices, diffusive risk, jump risk, riskless rate.

JEL Classification Codes: D51, D82, G12.

## 1 Introduction

It is well known that moral hazard is one of the most serious problems in finance. There exists a huge literature on moral hazard in corporate finance (e.g. Tirole (2006, Section 3.2)). In a typical moral-hazard model, the probability of success of a firm's project depends on its manager's hidden effort, which is incontractible. An investor then needs to give the manager an incentive to avoid his opportunistic misbehavior. Therefore, the moral hazard distorts optimal risk sharing and allocation. Still, such micro effects are not all the distortion causes in the financial world. The moral hazard twists the investor's marginal utility (i.e., pricing kernel) as well and thus affects the valuation of not only the firm but also all other financial assets in macro markets. How much return on investment does the investor demand to compensate for a loss caused by moral hazard?

Surprisingly, however, there have been not so many studies of such valuation of moral hazard in asset pricing and financial engineering (e.g. fixed-income investment, the term structure of interest rates, corporate risk management, and actuarial insurance). A notable exception is Ou-Yang (2005), who studies equilibrium asset pricing in the presence of moral hazard on the assumption of the exponential utility function and an exogenous constant riskless rate. Still, Ou-Yang (2005, p.1283) himself discusses the importance of incorporating more general utility functions such as the power utility function. In addition, the assumption of the exogenous riskless rate seems to have limited its applicability to financial practices.

The purpose of this paper is to provide an explicit asset-pricing formula in the presence of moral hazard on the assumption of the power utility function and an endogenous riskless rate. Specifically, we incorporate moral hazard into a continuous-time generalequilibrium exchange economy. We solve the problem of the investor's optimal consumption/wealth allocation in financial markets under two types of risk, i.e., regular (diffusive) risk and rare-event (negative jump) risk, when the process of production is subject to a firm manager's moral hazard. We then obtain explicitly an equilibrium state price, which can be used for pricing any kinds of securities and loans in the presence of moral hazard.

The more specific model setup is as follows. There exist two players: the representative investor and a representative firm (i.e., the firm manager) over [0, T]. Both players rank their consumption based on time-separable utility with common time preference. The investor's instantaneous utility function is of a constant relative risk aversion (CRRA) type while the firm's is of a log type. The firm produces a single non-storable consumption good over time and shares it with the investor. No productive resources are utilized: the production is an endowment, which stands for real macro outcomes (GDP). The firm can control ex-post the probability measure by incurring effort costs: moral hazard is defined here as the firm's ex-post, costly, strategic change of measure that is incontractible. The endowment process is thus subject to moral hazard. The investor controls ex-post optimally her consumption/wealth with access to financial markets while designing ex-ante optimally an incentive contract for the firm so that the manager does not expect better rewards from his opportunistic misbehavior. We look at the macroeconomic effect of moral hazard.

This paper consequently provides an explicit asset-pricing formula under the moral-hazard problem. We obtain equilibrium state prices (i.e., riskless rate and the market prices of diffusive risk and jump risk) and make clear the structural effect of moral hazard on

<sup>\*</sup>This is a short version of our paper, namely Misumi et al. (2014), modified for the 2014 JARIP annual meeting proceedings. For the details of the paper, see the full version. Corresponding author: Hisashi Nakamura, Graduate School of Commerce and Management, Hitotsubashi University, 2-1 Naka, Kunitachi, Tokyo 186-8601, Japan. Phone: +81-42-580-8826. Fax: +81-42-580-8747. Email: hisashi.nakamura@r.hit-u.ac.jp. We would like to thank Jakša Cvitanić for his very helpful advice at an early stage of this research. We are also thankful to Masaaki Fukasawa, Chiaki Hara, Tokuo Iwaisako, Hong Joo Jung, Masaaki Kijima, Yukinobu Kitamura, Katsumasa Nishide, Kazuhiko Ohashi, Hideyuki Takamizawa, and participants at APRIA Moscow 2014 for their valuable comments.