PHASE-TYPE APPROXIMATION OF THE GERBER-SHIU FUNCTION

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ABSTRACT. The Gerber-Shiu function provides a way of measuring the risk of an insurance company. It is given by the expected value of a function that depends on the ruin time, the deficit at ruin, and the surplus prior to ruin. Its computation boils down to the evaluation of the overshoot/undershoot distributions of the surplus process at ruin. In this paper, we approximate it in a closed form by fitting the underlying process by phase-type Lévy processes.

Key words: ruin theory, Gerber-Shiu functions, phase-type distributions; spectrally negative Lévy processes; scale functions

1. INTRODUCTION

The fundamental objective of the actuarial ruin theory is to measure the vulnerability to insolvency. Typically, the surplus of an insurance company is modeled by a stochastic process, and ruin occurs at the first time it goes below a certain threshold. A most classical and important quantity of interest is the ruin probability, and the Gerber-Shiu function is its generalization; it is given as an expected discounted value of the cost function that is dependent on the ruin time, the deficit at ruin and the surplus prior to ruin. The evaluation of the Gerber-Shiu function involves that of the overshoot and undershoot distributions at the first down-crossing time that do not admit explicit expressions. Hence its computation is in general a challenging task.

In ruin theory, the surplus process is commonly modeled by a stochastic process with downward jumps. Due to the premiums received from the insured persons, the surplus tends to increase constantly. On the other hand, it experiences sudden downward jumps due to the insurance payments. The classical Cramér-Lundberg model uses a compound Poisson process with downward jumps. Its generalization called the Sparre-Anderson model modifies it by allowing the arrival of the claims to follow a general renewal process.

In the last decade, significant progress has been made in insurance mathematics and ruin theory thanks to the development of the theory of Lévy processes [7, 16]. In particular, many results on the Cramér-Lundberg model have been generalized for a general spectrally negative Lévy process, or the Lévy process with only downward jumps; see, e.g., [3, 5, 6, 19]. This generalization enables one to construct more realistic models; one can, for example, introduce noise by including Brownian motion and/or infinitesimal jumps of infinite activity/variation.

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